Basic rules and principles

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We lay out the fundamental rules of computation(let's call them the First Principle for Algebra!), some of which will evolve as we expand the number system. Pretty boring, but we have to obey them.

1. We have two operations while counting natural numbers: $+,\ \times$

   The affiliated rules are:

   (1.1). Commutative rule:

$$a + b = b + a;\ \ a \times b = b \times a.$$

   (1.2). Associative rule:

$$(a + b) + c = a + (b + c);\ \ (a \times b) \times c = a \times (b \times c).$$

   (1.3). Distributive rule:

$$a \times (b + c) = a \times b + a \times c.$$

The rules hold for all complex numbers!

2. Exponent operation

Counting decimal numbers leads to the exponent operation.

$$a^n = \underbrace{a \times a \times \ldots \times a}_{n\ copies}$$

The affiliated rules (these are the rules that will evolve as exponents change from integers to fraction, to real numbers, and to complex numbers) for nonzero bases a, b and integer exponents n, m are

(2.1). Product rule:

$$a^n \times a^m = a^{n+m}.$$

(2.2). Power of a power rule:

$$(a^n)^m = a^{nm}.$$

(2.3). Power of a product rule:

$$(ab)^n = a^n b^n.$$

3. Equations and inequalities

When we deal with equations and inequalities, we have the following rules (we need to find solutions and verify these are the only ones!).

For equations

(3.1). (Invariant under addition): For any number C,

$$A = B \Leftrightarrow A + C = B + C.$$

(3.2). (Invariant under multiplication): For any number $C \neq 0$,

$$A = B \Leftrightarrow A \times C = B \times C.$$

(3.3). (Symmetry): $A = B \Leftrightarrow B = A.$

(3.4). (Transition)： If $A = B$, and $B = C$, then $A = C$.

For inequalities

(3.5). (Invariant under addition)：For any number C,

$$A > B \Leftrightarrow A + C > B + C.$$

(3.6a). (Invariant under positive number multiplication): For any number $C > 0$,

$$A > B \Leftrightarrow A \times C > B \times C.$$

(3.6b). (Reverse order under negative number multiplication):

For any number $C < 0$,

$$A > B \Leftrightarrow A \times C < B \times C.$$

(3.7). (Symmetry)： $A > B \Leftrightarrow B < A$.

(3.8). (Transition)： If $A > B$, and $B > C$, then $A > C$.

Now, let's start the Algebra journey!